

## MATH 204 C03 - SUPPLEMENT 1

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Supplements are extra problems. While not required, they do provide at least one tangible benefit if turned in with correct work:

As with theorems from the book and what is presented in class, any work a student in section C03 completes successfully in an offered supplement may be considered “known” for homework problems. This may allow in some cases for easier and shorter solutions. *Indicate at the beginning of an assignment that you are using this result (such as “using supplement 1”).*

This benefit is available to any applicable student who submits a correct proof of the stated problems. Such work must be pledged with the honor code. Except for your instructor and the textbook, **no collaboration is allowed** and **no other sources may be referenced**. The instructor may revoke this benefit at his discretion.

### NOT A MATH QUESTION

Suppose Jon is your instructor for this course. Also, assume that Jon already has an office hour each Wednesday from 2-3pm in his office, Fine 1104. Which of the following two times would you prefer for Jon’s second office hour: Fridays 10-11am, or Thursdays 4-5pm?

### MATH QUESTION

Suppose  $\mathcal{V}$  is a vector space and  $\mathcal{S}$  is a finite set of elements in  $\mathcal{V}$ . Let  $\mathcal{S}'$  be the result of performing one of the following operations:

- A. Multiplying an element of  $\mathcal{S}$  by a non-zero scalar.
- B. Adding a non-zero scalar multiple of one element of  $\mathcal{S}$  to another.

Prove the following:

- (1)  $\text{span } \mathcal{S} = \text{span } \mathcal{S}'$ .
- (2)  $\mathcal{S}$  is linearly independent if and only if  $\mathcal{S}'$  is linearly independent.
- (3) If  $\mathcal{T}$  is obtained from  $\mathcal{S}$  by a sequence of actions of type A and B, then the above two statements hold for  $\mathcal{T}$  and  $\mathcal{S}$  as well.

*(Note that a set can be written in any order, so these facts are trivially true for the “other” elementary operation of switching the order of two elements.)*